Time-varying risk premium in large cross-sectional equity datasets

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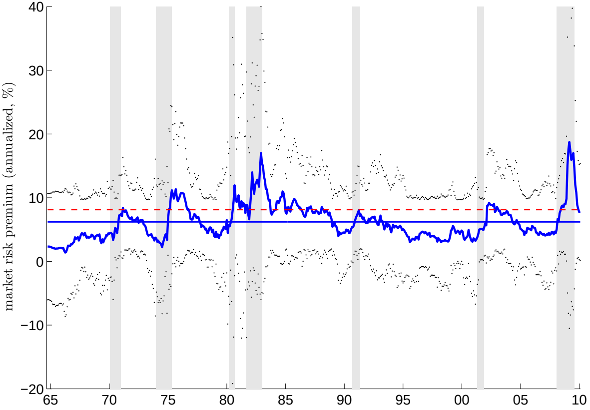
Goal of the paper

- Analysis of time-varying behaviour of risk premia in large equity datasets.



Two-pass regression methodology

$$R_{i,t} = a_i + b_i' f_t + \varepsilon_{i,t},\ \ t = 1, ..., T,\ \ i = 1, ..., n$$

$$E\left[R_{i,t}\right] = b_i' \lambda$$

Two-pass methodology
(Black-Jensen-Scholes (1972), Fama-MacBeth (1973)):

1. time series OLS regression to estimate the factor loadings b_i;
2. cross-sectional OLS regression to estimate the vector of risk premia λ.

Usual setting:

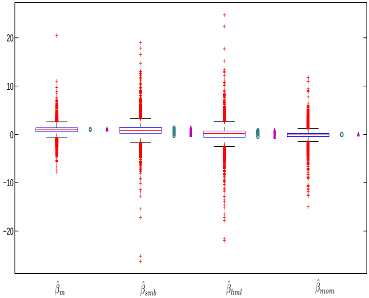
- time-invariant linear factor models of asset returns;
- portfolios with large T and fixed n (balanced panel).

This paper:

- time-varying linear factor models of asset returns;
- individual stocks with large T and large n ($n >> T$ and unbalanced).

Individual stocks versus portfolios

Estimated factor loadings for individual stocks (box-plots), for 25 (circles) and 100 FF portfolios (triangles)



Sorting and pooling stocks into portfolios distorts information.

Data-snooping bias (Lo-MacKinlay (1990)).

Ang-Liu-Schwarz (2008), Lewellen-Nagel-Shanken (2010), Berk (2000)

Building blocks of the thesis

1. Derivation of no-arbitrage pricing restrictions
   - In a **large economy** (continuum of assets)
     Hansen-Richard (1987), Al-Najjar (1995, 1998)
   - With an **approximate factor structure** for excess returns
     Chamberlain-Rothschild (1983), Al-Najjar (1999)
   - With **conditional** factor models for excess returns
     Ferson-Harvey (1991,1999), Ferson-Schadt (1996), Ghysels (1998), Jagannathan-Wang (1996), and Petkova-Zhang (2005)

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     Ferson-Harvey (1991,1999), Ferson-Schadt (1996), Ghysels (1998), Jagannathan-Wang (1996), and Petkova-Zhang (2005)
2. A new two-pass cross-sectional estimator of the risk premia
   - **Large unbalanced panel** of returns
   - Large-sample properties with double asymptotics: $\mathbf{n}, \mathbf{T} \to \infty$
     Bai-Ng (2002, 2006), Stock-Watson (2002), Bai (2003, 2009), Forni-Hallin-Lippi-Reichlin (2000, 2004, 2005), and Pesaran (2006)
   - Comparison with the classical framework:
     balanced panel and $T \to \infty$ with n fixed
     Shanken (1985,1992), Jagannathan-Wang (1998), Kan-Robotti-Shanken (2009), and Shanken-Zhou (2007)

3. Test of the asset pricing restrictions
   - Based on the **cross-sectional SSR**
     Gibbons-Ross-Shanken (1985)
   - Relation to the coefficient of determination R^2 of cross-sectional regression
     Lewellen-Nagel-Shanken (2009), and Kan-Robotti-Shanken (2009)

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4. Empirical analysis comparing results with CRSP **individual stock returns** and Fama-French 25 portfolios
   - Use of individual stocks versus portfolios
     Litzenberger-Ramaswamy (1979), Berk (2000), Ang-Liu-Schwarz (2008), and Avramov-Chordia (2006)
   - Risk premia estimates disagree between individual stocks and portfolios

Outline of the presentation

- Introduction ✓
- Conditional factor model
  - Model setting
  - Functional specification of time-varying coefficients
  - Estimation of betas and risk premia
  - Testing of the asset pricing restrictions
- Empirical results
- Conclusions

Conditional factor model: Model setting

Excess returns generation and asset pricing restrictions:

The excess return $R_t(\gamma)$ of asset $\gamma \in [0,1]$ at date $t = 1, 2, ...,$ satisfies

$$R_t(\gamma) = \beta_t(\gamma)' x_t + \varepsilon_t(\gamma), \tag{1}$$

where:

- $x_t = (1, f_t')'$ and f_t is the $K \times 1$ random vector of observable factors;
- $\beta_t(\gamma) = \left(a_t(\gamma), b_t(\gamma)'\right)'$ contains time-varying coefficients;
- $\varepsilon_t(\gamma)$ is a random vector of error terms s.t. $E\left[\varepsilon_t(\gamma) | \mathcal{F}_{t-1}\right] = 0$ and $Cov\left[\varepsilon_t(\gamma), f_t | \mathcal{F}_{t-1}\right] = 0$ for any $\gamma \in [0,1]$.

(Hansen-Richard (1987))

Assumption 1:

Approximate factor structure: (Chamberlain-Rothschild (1983)) conditional var-cov matrix $\Sigma_{\varepsilon,t,n} = \left[Cov\left[\varepsilon_t(\gamma_i), \varepsilon_t(\gamma_j) | \mathcal{F}_{t-1}\right]\right]_{i,j}$ for $i, j = 1, ..., n$ is s.t. $n^{-1} eig_{max}\left(\Sigma_{\varepsilon,t,n}\right) \xrightarrow{L^2} 0$ as $n \to \infty$, for a.e. sequences (γ_i) in $[0,1]^{\infty}$;
No asymptotic arbitrage opportunities: there are no portfolios that approximate arbitrage opportunities when the number of assets increases.

Proposition 1: Asset pricing restriction

There exists a unique vector $\nu_t \in \mathbb{R}^K$ such that

$$a_t(\gamma) = b_t(\gamma)' \nu_t \qquad \left(\text{i.e., } E\left[R_t(\gamma) | \mathcal{F}_{t-1}\right] = b_t(\gamma)' \lambda_t\right) \tag{2}$$

for almost all $\gamma \in [0,1]$, where $\lambda_t = \nu_t + E\left[f_t | \mathcal{F}_{t-1}\right]$ is the vector of time-varying risk premia.

Large economy with a continuum of assets:

$\Rightarrow$ derivation of an **empirically testable exact pricing restriction**.

$\Rightarrow$ **robustness** of factor structures to asset **repackaging** (Al-Najjar (1999)).

Unbalanced nature of the panel:

$I_t(\gamma)$ admits value 1 if the return of asset γ is observable at date t, and 0 otherwise (Connor-Korajczyk (1987)).

The sampling scheme:

A sample of n assets is obtained by drawing i.i.d. indices γ_i according to a probability distribution G on $[0,1]$.

$\Rightarrow$ **cross-sectional limits exist and are invariant to reordering of assets.**

$\Rightarrow$ sample of n assets and T observations of excess returns $R_{i,t} = R_t(\gamma_i)$, $I_{i,t} = I_t(\gamma_i)$, $\varepsilon_{i,t} = \varepsilon_t(\gamma_i)$ and $\sigma_{ij,t} = E\left[\varepsilon_{i,t}\varepsilon_{j,t} | \mathcal{F}_{\underline{t}}, \gamma_i, \gamma_j\right]$ for $i = 1, ..., n$ and $t = 1, ..., T$.

$\Rightarrow$ **random coefficient panel** model with $\beta_{i,t} = \beta_t(\gamma_i)$.

Functional specification of time-varying coefficients

Information set $\mathcal{F}_{t-1}$ contains lagged observations of:

- $Z_t \in \mathbb{R}^p$, vector of common instruments:
  - the constant and the observable factors f_t,
  - additional observable variables Z_t^*.
- $Z_{i,t} \in \mathbb{R}^q$, vector of asset-specific instruments:
  - firm characteristics,
  - stocks returns.

Assumption 2:

Factor loadings: $b_t(\gamma) = B(\gamma) Z_{t-1} + C(\gamma) Z_{t-1}(\gamma)$, where $B(\gamma) \in \mathbb{R}^{K \times p}$ and $C(\gamma) \in \mathbb{R}^{K \times q}$, for any $\gamma \in [0,1]$ and $t = 1, 2, \ldots$;

Risk premia: $\lambda_t = \Lambda Z_{t-1}$, where $\Lambda \in \mathbb{R}^{K \times p}$, for any t;

Factors: $E[f_t | \mathcal{F}_{t-1}] = F Z_{t-1}$, where $F \in \mathbb{R}^{K \times p}$, for any t.

Assumption 2 and Proposition 1 imply:

$$a_t(\gamma) = Z'_{t-1}B(\gamma)'(\Lambda - F)Z_{t-1} + Z_{t-1}(\gamma)'C(\gamma)'(\Lambda - F)Z_{t-1}.$$

- The **conditional factor model** (1), for the sample observations, becomes

$$R_{i,t} = \beta_i' x_{i,t} + \varepsilon_{i,t}, \tag{3}$$

  where:
  - regressor $x_{i,t}$ involves cross-terms of instruments Z_{t-1}, $Z_{i,t-1}$ and f_t;
  - time-invariant parameters $\beta_i = \left(\beta'_{1,i}, \beta'_{2,i}\right)'$ are (unconditional) transformations of matrices B_i, C_i, Λ and F.
- The **asset pricing restriction** (2) implies the parameter restriction

$$\beta_{1,i} = \beta_{3,i}\nu, \tag{4}$$

  where:
  - $\beta_{3,i}$ is a trasformation of matrices B_i and C_i;
  - $\nu = vec\left[\Lambda' - F'\right]$.

Estimation of betas and risk premia

1. ***Time series regression*** *for the first pass:*

$$\hat{\beta}_i = \left(\sum_t I_{i,t} x_{i,t} x'_{i,t} \right)^{-1} \sum_t I_{i,t} x_{i,t} R_{i,t}, \; i = 1, ..., n.$$

Problem: If $T_i = \sum_t I_{i,t}$ is small, the inversion of $\hat{Q}_{x,i} = \frac{1}{T_i} \sum_t I_{i,t} x_{i,t} x'_{i,t}$ can be unstable.

Idea: Apply a **trimming approach**:

$$\mathbf{1}_i^{\chi} = \mathbf{1} \left\{ CN\left(\hat{Q}_{x,i}\right) \leq \chi_{1,T}, \tau_{i,T} \leq \chi_{2,T} \right\},$$

with $\chi_{1,T} > 0$ and $\chi_{2,T} > 0$ and where $CN\left(\hat{Q}_{x,i}\right) = \sqrt{\frac{eig_{max}\left(\hat{Q}_{x,i}\right)}{eig_{min}\left(\hat{Q}_{x,i}\right)}}$ is the condition number of $\hat{Q}_{x,i}$ (Greene (2008)), and $\tau_{i,T} = T/T_i$.

2. ***Cross-sectional WLS regression*** *for the second pass:*

$$\hat{\nu} = \left(\sum_i \hat{\beta}'_{3,i} \hat{w}_i \hat{\beta}_{3,i} \right)^{-1} \sum_i \hat{\beta}'_{3,i} \hat{w}_i \hat{\beta}_{1,i},$$

where $\hat{w}_i = \mathbf{1}_i^{\chi} \left(diag \left[\hat{v}_i \right] \right)^{-1}$ and $\hat{v}_i$ is a consistent estimator of $v_i = AsVar \left[\sqrt{T} \left(\hat{\beta}_{1,i} - \hat{\beta}_{3,i} \nu \right) \right]$.

The **estimator of time-varying risk premia** is

$$\hat{\lambda}_t = \hat{\Lambda} Z_{t-1},$$

where $\hat{\Lambda}$ is deduced by

$$vec \left[\hat{\Lambda}' \right] = \hat{\nu} + vec \left[\hat{F}' \right],$$

and $\hat{F}$ is the estimator of F in the SUR regression: $f_t = F Z_{t-1} + u_t$.

Large sample properties

Asymptotic scheme: ***simultaneous double asymptotic***

$n, T \to \infty$ such that $n = T^{\bar{\gamma}}$ with $\bar{\gamma} > 0$.

Assumption 3: Heteroschedasticity and cross-sectional dependence

a) $E\left[\varepsilon_{i,t} \middle| \left\{\varepsilon_{j,\underline{t-1}}, \gamma_j, j = 1, ..., n\right\}, \mathcal{F}_{\underline{t}}\right] = 0$, *with* $\varepsilon_{j,\underline{t-1}} = \{\varepsilon_{j,t-1}, \varepsilon_{j,t-2}, \cdots\}$;

b) $M^{-1} \leq E\left[\varepsilon_{i,t}^2 | \mathcal{F}_t, \gamma_i\right] = \sigma_{ii,t} \leq M, \quad i = 1, ..., n$ *for a constant* $M < \infty$;

c) $E\left[\frac{1}{n}\sum_{i,j} E\left[\left|\sigma_{ij,t}\right|^2 |\gamma_i, \gamma_j\right]\right]^{1/2} \leq M,$ *with* $\sigma_{ij,t} = E\left[\varepsilon_{i,t}\varepsilon_{j,t}|\mathcal{F}_t, \gamma_i, \gamma_j\right].$

Assumption 3 accommodates **non Gaussian**,
conditionally heteroschedastic,
weakly **serially and cross-sectionally dependent** error terms.

Proposition 2: Asymptotic distribution

As $n, T \to \infty$ such that $n = o(T^3)$, estimators $\hat{\nu}$, $\hat{\Lambda}$ and $\hat{\lambda}_t$ are consistent and asymptotically normal:

a) $\sqrt{nT}\left(\hat{\nu} - \nu - \frac{1}{T}\hat{B}_\nu\right) \Rightarrow N\left(0, \Sigma_\nu\right)$, *where* $\hat{B}_\nu / T$ *is a bias term;*

b) $\sqrt{T} vec\left[\hat{\Lambda}' - \Lambda\right] \Rightarrow N\left(0, \Sigma_\Lambda\right)$, *where*

$$\Sigma_\Lambda = \left(I_K \otimes Q_z^{-1}\right) \Sigma_u \left(I_K \otimes Q_z^{-1}\right),$$

with $Q_z = E\left[Z_t Z_t'\right]$ *and* $\Sigma_u = E\left[u_t u_t' \otimes Z_{t-1} Z_{t-1}'\right]$;

c) $\sqrt{T}\left(\hat{\lambda}_t - \lambda_t\right) \Rightarrow N\left(0, H_{t-1}\Sigma_\Lambda H_{t-1}'\right)$, *where* H_{t-1} *is a trasformation of* Z_{t-1}.

Estimation of ν does not affect accuracy of risk premia estimates.

Properties:

- Estimators $\hat{\nu}$, $\hat{\Lambda}$ and $\hat{\lambda}_t$ feature **different convergence rates** $\sqrt{nT}$ and $\sqrt{T}$.
- Bias term $\hat{B}_\nu / T$ is induced by the Error-in-Variable (EIV) problem.

Time-invariant case ($Z_t = 1$ and $Z_{i,t} = 0$):

- $\hat{\lambda} = \hat{\nu} + \frac{1}{T}\sum_t f_t$ and $\hat{\nu} = \left(\sum_i \hat{w}_i \hat{b}_i \hat{b}_i'\right)^{-1} \sum_i \hat{w}_i \hat{b}_i \hat{a}_i$ with $\hat{w}_i = \hat{v}_i^{-1}$;
- for $n, T \to \infty$, $\sqrt{T}\left(\hat{\lambda} - \lambda\right) \Rightarrow N\left(0, \Sigma_f\right)$;
- for **fixed** n, $T \to \infty$, $\sqrt{T}\left(\hat{\lambda} - \lambda\right) \Rightarrow N\left(0, \Sigma_f + \frac{1}{n}\Sigma_\nu\right)$ (Shanken (1992), Jagannathan-Wang (1998)).

Link with the well-known incidental parameters problem in the fixed effects nonlinear panel literature

Write the time-invariant factor model, with asset pricing restriction $a_i = b_i'\nu$, as:

$$R_{i,t} = b_i'(f_t + \nu) + \varepsilon_{i,t},$$

where the b_i are the individual effects and ν is the common parameter.

Hahn-Kuersteiner (2002), Hahn-Newey (2004)): $y_{i,t} \sim h(\cdot; b_i, \nu)$

- Similar type of analytical bias correction for the estimator of ν.
- Same condition $n = o(T^3)$ for the asymptotic analysis.
- However, our setting is semi-parametric and accommodates cross-sectional dependence.

Estimation of asymptotic variance Σ_ν

Problem: Σ_ν involves the double sum

$$S_{v_3} = \lim_{n\to\infty} E\left[\frac{1}{n}\sum_{i,j}\frac{\tau_i\tau_j}{\tau_{ij}}\left(Q_{x,i}^{-1}S_{ij}Q_{x,j}^{-1}\right)\otimes v_{3,i}v_{3,j}'\right],$$

over $S_{ij} = E[\varepsilon_{i,t}\varepsilon_{j,t}x_{i,t}x_{j,t}'|\gamma_i,\gamma_j]$, where $v_{3,i} = vec[\beta_{3,i}'w_i]$.

Plugging-in $\hat{S}_{ij} = \frac{1}{T_{ij}}\sum_t I_{i,t}I_{j,t}\hat{\varepsilon}_{i,t}\hat{\varepsilon}_{j,t}x_{i,t}x_{j,t}'$ leads to divergent accumulation of statistical errors.

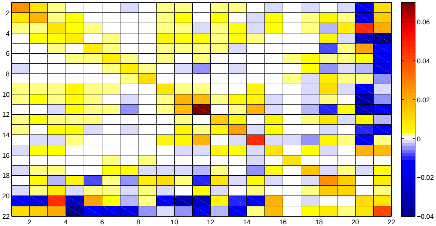
Idea:

Assume a sparsity structure for the S_{ij} and use a thresholded estimator (Bickel-Levina (2008), Fan-Liao-Mincheva (2011))

$$\tilde{S}_{ij} = \hat{S}_{ij}\mathbf{1}_{||\hat{S}_{ij}||\geq\kappa}.$$

Sparsity condition is applied on the error terms and *not* on the excess returns!

Average correlation matrix of residuals of individual stocks grouped by industrial sectors (Ferson-Harvey (1999)).



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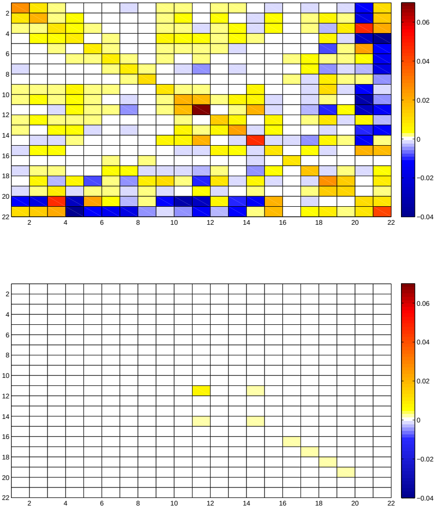


Testing of the asset pricing restriction

$$\mathcal{H}_0\text{: there exists } \nu \in \mathbb{R}^{pK} \text{ such that } \beta_1(\gamma) = \beta_3(\gamma)\nu, \\ \text{for almost all } \gamma \in [0,1].$$

- The statistic is $\hat{\xi}_{nT} = T\sqrt{n}\left(\hat{Q}_e - \frac{1}{T}\hat{B}_\xi\right)$, where
  - $\hat{Q}_e = \frac{1}{n}\sum_i \hat{e}_i'\hat{w}_i\hat{e}_i$, with $\hat{e}_i = \hat{\beta}_{1,i} - \hat{\beta}_{3,i}\hat{\nu}$, is the cross-sectional weighted SSR (Gibbons-Ross-Shanken (1989));
  - $\hat{B}_\xi = 0.5p(p+1) + pq$ is the recentering term.

Proposition 3: Asymptotic distribution of the test statistic under $\mathcal{H}_0$

Under $\mathcal{H}_0$, we have $\tilde{\Sigma}_\xi^{-1/2}\hat{\xi}_{nT} \Rightarrow N(0,1)$, as $n, T \to \infty$ such that $n = o(T^2)$, where $\tilde{\Sigma}_\xi$ is an estimator of the asymptotic variance that involves the thresholded estimator $\tilde{S}_{ij}$.

- More restrictive condition on the relative rate of n and T wrt Prop. 2.

Data description

Base assets:

- 9,936 stocks with monthly returns from Jul1964 to Dec2009 after merging CRSP and Compustat databases;
- 25 and 100 Fama-French (FF) monthly portfolios returns.

Factors:

- $f_t = \left(r_{m,t}, r_{smb,t}, r_{hml,t}, r_{mom,t} \right) = (\text{market, size, value, momentum})$.

Instrumental variables:

- common variables $Z_t = (1, Z_t^*)'$:
  - term spread: difference between yields on 10-year Treasurys and 3-month T-bills;
  - default spread: yield difference between Moody's Baa and Aaa-rated corporate bonds.
- firm characteristics $Z_{i,t}$:
  - book-to-market equity.

Estimated risk premia for the time-invariant models

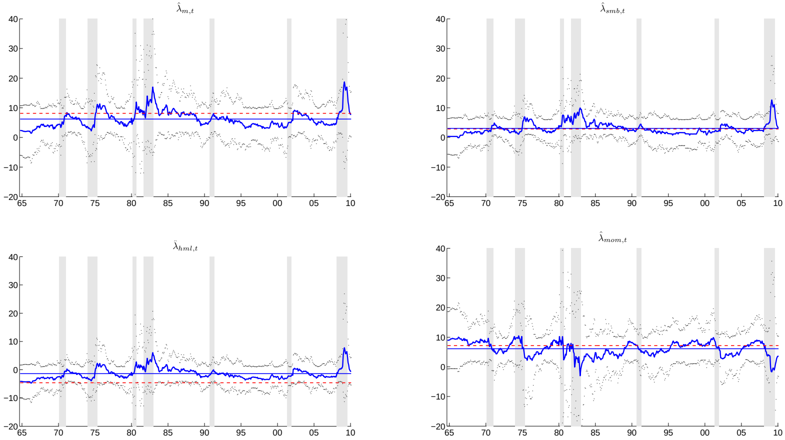
	Stocks ($n = 9,936$, $n^{\chi} = 9,902$)		Portfolios ($n = n^{\chi} = 25$)	
	bias corrected estimate (%)	95% conf. interval	point estimate (%)	95% conf. interval
		Four-factor model		
λ_m	8.14	(3.26, 13.02)	5.70	(0.73, 10.67)
λ_{smb}	2.86	(-0.50, 6.22)	3.02	(-0.48, 6.51)
λ_{hml}	-4.60	(-8.06, -1.14)	4.81	(1.21, 8.41)
λ_{mom}	7.16	(2.56, 11.75)	34.03	(9.98, 58.07)
		Fama-French model		
λ_m	7.77	(2.89, 12.65)	5.04	(0.11, 9.97)
λ_{smb}	2.64	(-0.72, 5.99)	3.00	(-0.42, 6.42)
λ_{hml}	-5.18	(-8.65, -1.72)	5.20	(1.66, 8.74)
		CAPM		
λ_m	7.42	(2.54, 12.31)	6.98	(1.93, 12.02)

Estimated ν for the time-invariant models

	Stocks ($n = 9,936$, $n^{\chi} = 9,902$)		Portfolios ($n = n^{\chi} = 25$)	
	bias corrected estimate (%)	95% conf. interval	point estimate (%)	95% conf. interval
		Four-factor model		
ν_m	3.29	(2.88, 3.69)	0.85	(-0.10, 1.79)
ν_{smb}	-0.41	(-0.95, 0.13)	-0.26	(-1.24, 0.72)
ν_{hml}	-9.38	(-10.12, -8.64)	0.03	(-0.95, 1.01)
ν_{mom}	-1.47	(-2.86, -0.08)	25.40	(1.80, 49.00)
		Fama-French model		
ν_m	2.92	(2.48, 3.35)	0.18	(-0.51, 0.87)
ν_{smb}	-0.63	(-1.11, -0.15)	-0.27	(-0.93, 0.40)
ν_{hml}	-9.96	(-10.62, -9.31)	0.41	(-0.32, 1.15)
		CAPM		
ν_m	2.57	(2.17, 2.97)	2.12	(0.85, 3.40)

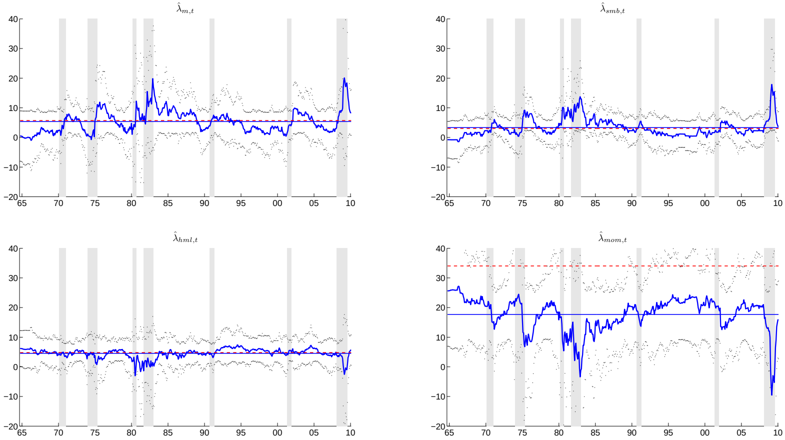
Paths of estimated risk premia with $n = 9,936$

Annualized % risk premia for individual stocks



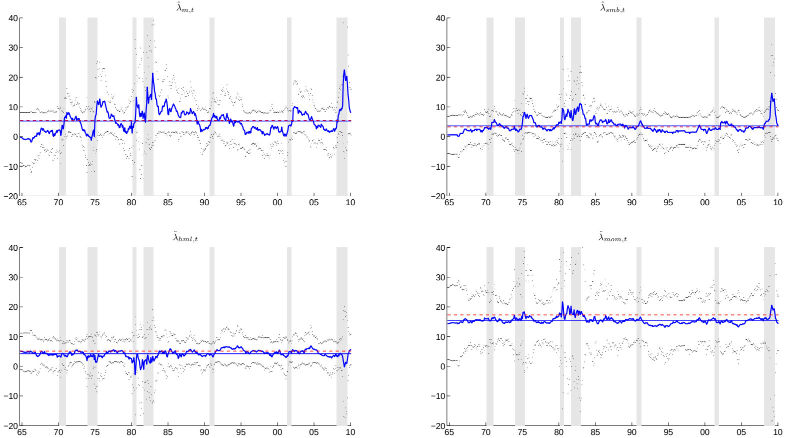
Paths of estimated risk premia with $n = 25$

Annualized % risk premia for Fama-French portfolios



Paths of estimated risk premia with $n = 100$

Annualized % risk premia for Fama-French portfolios



Effects of $vec\left[F'\right]$ and ν on time-varying risk premia

		$vec\left[F'\right]$	ν $(n=9,936)$	ν $(n=25)$
m	*const*	4.8322 (0.2653, 9.3990)	1.3744 (0.6791, 2.0697)	0.5251 (−0.4704, 1.5206)
	ds_{t-1}	3.0353 (−2.6803, 8.7509)	−0.6032 (−1.2964, 0.0899)	−0.2916 (−1.1614, 0.5782)
	ts_{t-1}	1.8677 (−2.8399, 6.5754)	−0.9254 (−1.5914, −0.2593)	0.0828 (−0.6660, 0.8316)
smb	*const*	3.2739 (0.0410, 6.5067)	−0.2130 (−0.8933, 0.4674)	0.0607 (−0.9898, 1.1112)
	ds_{t-1}	2.5468 (−0.5998, 5.6934)	−0.5948 (−1.1622, −0.0273)	0.4134 (−0.6129, 1.4397)
	ts_{t-1}	0.2855 (−2.6271, 3.1982)	−0.2157 (−0.7584, 0.3269)	−0.1966 (−0.9679, 0.5746)
hml	*const*	4.7772 (1.7905, 7.7639)	−6.1642 (−6.8891, −5.4393)	−0.2267 (−1.3134, 0.8601)
	ds_{t-1}	−1.7898 (−5.5963, 2.0167)	3.5981 (2.8651, 4.3311)	0.2187 (−1.0354, 1.4728)
	ts_{t-1}	0.8933 (−2.2598, 4.0465)	−0.4292 (−1.0310, 0.1726)	−0.0073 (−0.8758, 0.8612)
mom	*const*	8.6543 (−4.2482, 13.0605)	−2.5592 (−3.4360, −1.6825)	9.0179 (0.4373, 17.5986)
	ds_{t-1}	−7.3714 (−14.6656, −0.0771)	6.0148 (5.0903, 6.9394)	1.9403 (−5.9930, 9.8736)
	ts_{t-1}	1.5804 (−2.8226, 5.9833)	−3.2960 (−4.0400, −2.5519)	−2.5080 (−9.9800, 4.9641)

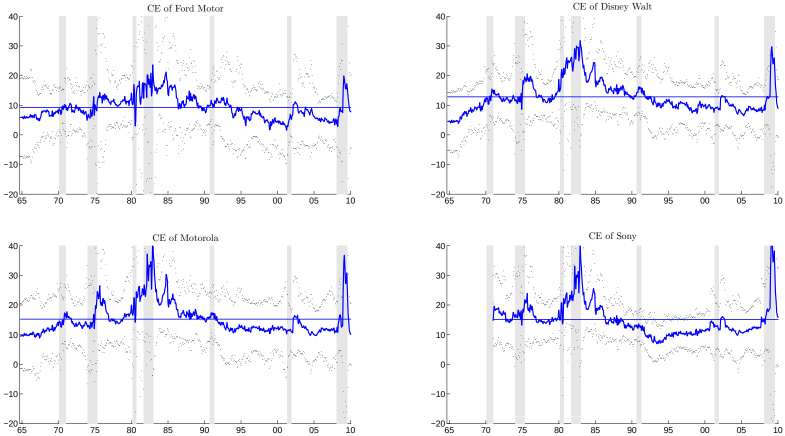
Time variation tests

$\mathcal{H}_0^F$: $Avec\left[F'\right] = 0$	$\mathcal{H}_0^\nu$: $A\nu = 0$	
	Stocks ($n = 9,936$)	Portfolios ($n = 25$)
11.8765 (0.1570)	389.27 (0.0000)	1.5566 (0.9920)

- Matrix A is a selection matrix for the components of $vec\left[F'\right]$ and ν corresponding to the effects of the instruments.
- For individual stocks, we reject time-invariance of risk premia implied by the rejection of $\mathcal{H}_0^\nu$.
- The aggregation in the 25 FF portfolios completely masks the time variation of the risk premia.

Paths of estimated cost of equity

$$\text{Cost of equity: } CE_{i,t} = r_{f,t} + b_{i,t}' \lambda_t$$



Test results for asset pricing restriction in the time-invariant model

	$\mathcal{H}_0: \ a(\gamma) = b(\gamma)' \nu$		$\mathcal{H}_0: \ a(\gamma) = 0$	
	$n^\chi = 1,400$ $N(0,1)$	$n = 25$ χ^2_{n-K}	$n^\chi = 1,400$ $N(0,1)$	$n = 25$ χ^2_n
	Four-factor model			
Test statistic	2.0088	35.2231	19.1803	74.9100
p-value	0.0223	0.0267	0.0000	0.0000
	Fama-French model			
Test statistic	2.9593	83.6846	28.0328	87.3767
p-value	0.0015	0.0000	0.0000	0.0000
	CAPM			
Test statistic	8.2576	110.8368	11.5882	111.6735
p-value	0.0000	0.0267	0.0000	0.0000

Test results for asset pricing restriction in the time-varying model

	$\mathcal{H}_0: \beta_1(\gamma) = \beta_3(\gamma)\nu$		$\mathcal{H}_0: \beta_1(\gamma) = 0$	
	$n^{\mathcal{X}} = 1,373$	$n = 25$	$n^{\mathcal{X}} = 1,373$	$n = 25$
	$N(0,1)$	$\frac{1}{n}\sum_j eig_j \chi_j^2$	$N(0,1)$	$\frac{1}{n}\sum_j eig_j \chi_j^2$
		Four-factor model		
Test statistic	3.2514	13.4815	3.8683	14.3080
p-value	0.0000	0.0000	0.0000	0.0000
		Fama-French model		
Test statistic	3.1253	15.7895	3.8136	15.9038
p-value	0.0000	0.0000	0.0000	0.0000
		CAPM		
Test statistic	1.7322	9.2934	1.7381	9.6680
p-value	0.0416	0.2076	0.0411	0.0000

Conclusions

Finance Theory:

- We derive empirically testable no-arbitrage restrictions in a multi-period conditional economy with a continuum of assets and an approximate factor structure.

Econometric Theory:

- Simple two-pass cross-sectional regressions allow us to estimate the time-varying risk premia implied by conditional linear asset pricing models using the returns of individual stocks.
- The risk premia estimator is consistent and asymptotically normal when $n, T \to \infty$.

Empirics:

- We observe a disagreement between the empirical results derived by sorting and pooling stocks into portfolios and by extracting the information directly from the individual stocks.